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COMMENT

Some speculations on the critical exponents and fractal dimensionalities relevant to realistic spin glasses

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Abstract. Some speculations on the critical exponents and fractal dimensionalities relevant to realistic spin glasses are presented.

Recently Chowdhury and Bhattacharjee (CB, 1984) have speculated on a relevant fractal dimensionality $D_f = 2$ for realistic spin glass alloys. Their speculation is based on the percolation model of spin glasses (Mookerjee and Chowdhury 1983) and experimental results on metallic spin glasses (Tholence 1980, Murani 1976). The percolation model, a resurrection of the old model of Wohlfarth (1977) incorporating the ideas of Smith (1975) and Abrikosov (1978), envisages the formation of clusters of dynamically correlated spins from a paramagnetic background as the temperature is lowered. The evolution of these clusters has been mapped by Mookerjee and Chowdhury (1983) onto a problem of percolation of overlapping spheres with randomly distributed centres and radii increasing with temperature according to a given prescription compatible with the 'locking' of two spins at sphere centres whenever their radii overlap. The percolation threshold is identified with the spin glass transition at that concentration of sphere centres.

Simple arguments by Hertz (1982, 1983) enable us to estimate the free energy barrier experienced by a cluster as it attempts to overturn thermally. The first idea (originally proposed by Palmer) is that a cluster can disorder by propagating a domain wall through it. This was the basis of equation (8) of CB. However, Hertz also argued that the barrier height is proportional both to the size and the mean square magnetisation seen in the experiment. This last dependence was ignored in the work of CB.

Near the percolation threshold T_g, the incipient infinite cluster (IIC) embedded in a $d = 3$ lattice is an object of fractal dimensionality $D_f = 2.5$ (Pandey and Stauffer 1983). It has a size ξ^{D_f}, where ξ is the relevant correlation length of the percolation process. Near T_g, the mean square magnetisation is expected to vanish linearly (Mookerjee 1978). Combining these ideas we get the barrier height $h(T)$ to be near T_g

$$h(T) = a(T - T_g)^{-x} \qquad \text{with } x = (D_f - 1)\nu - 1. \qquad (1)$$

Note that this estimate is different from the equation (8) of CB which entirely ignores the mean square magnetisation dependence.

The indices x and ν may be obtained from experiments on metallic spin glasses to which the sphere percolation model is applicable. The AC susceptibility experiments

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on CuMn (4.6% Mn) of Tholence (1980) yields an estimate of $x=0.8\pm0.1$ and $T_g\simeq 26$ K. The neutron scattering experiments of Murani (1976) on AuFe (10% Fe) yield an estimate of $\nu\simeq 1.2$. This is rather different from the usual exponent for nearest-neighbour percolation. This should not surprise us, as it is uncertain whether the sphere percolation belongs to the same universality class or not (Kertész 1981). To this date, I am not aware of any detailed work on exponents of the overlapping sphere percolation problem.

Notice that these estimates are compatible with equation (1) and our assumption that $D_f=2.5$. Substitution of these values in the equation (8) of CB gives a value of $D_f=1.67$ (and *not* 2 as stated by those authors). This led CB to speculate that one should use the backbone dimensionality. If we go back to the percolation model of Mookerjee and Chowdhury, we note that a cluster is defined to be a collection of spins which overturn together. The 'locking' criterion of Smith automatically leaves out those spins which are loosely coupled and overturn separately. It is this criterion which mathematically maps on to the sphere percolation problem. Given the model, the introduction of the idea of loose dead ends or cul de sacs does not seem to be consistent. Indeed, given the corrected equation (1) it is not necessary either. The normal $D_f=2.5$ works perfectly well.

This, of course, means that the exponent $\beta\simeq 1.37$ from our estimate of the probability of joining the infinite cluster $P(p)$ (Chowdhury and Mookerjee 1984) is incompatible with $D_f=d-\beta/\nu$. However, the exponent β was derived by us indirectly from only a few experimental points well away from T_g. As such that estimate is hardly reliable!

In all earlier work we confined ourselves to the low concentration regime, where even at T_g where the first frozen cluster of dynamically correlated spins forms, there is still no spontaneous magnetisation. This is because the random nature of the interaction ensures that the spins in the IIC are all frozen in random directions, so that the magnetisation is proportional to the square root of the cluster size. With increasing concentration transition to a ferromagnetic (long ranged ordered) state is still possible within this model. Suppose that the nearest-neighbour lattice distance is such that the RKKY interaction is ferromagnetic at this distance. As the concentration of spins increases, at high enough concentrations there may be sufficient unfrustrated nearest-neighbour spins on the IIC to lend it spontaneous magnetisation. The problem of spin glass to ferromagnetic transition at T_g with increasing concentration may be simulated by a percolation on the IIC. The indices at $p=p_c$, $T=T_g(p_c)$ could very well be anomalous (Pandey *et al* 1984, Havlin 1984). As reliable experimental data in this regime are not available this must remain a speculation at this stage.

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